1-1-1967

Analysis of Submergence in Flow Measuring Flumes

Gaylord V. V. Skogerboe

M. Leon Hyatt

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

ANALYSIS OF SUBMERGENCE IN FLOW MEASURING FLUMES

By Gaylord V. Skogerboe,[1] A. M. ASCE and M. Leon Hyatt,[2] A. M. ASCE

INTRODUCTION

Submerged flow exists in a measuring flume when a change in flow depth downstream from the flume causes a change in flow depth upstream for any particular constant value of discharge. When a change in tailwater depth does not affect the upstream depth, free flow exists. To evaluate the discharge under free-flow conditions, it is necessary to measure only a flow depth upstream from the contracted section (throat) of the flume, whereas two flow depths must be measured to evaluate the discharge under submerged-flow conditions. The two flow depths normally measured when submerged flow exists consist of the same upstream depth used for free flow and a depth measured in the throat, although this need not be the case as will be shown later.

Most of the earlier investigations regarding measuring flumes have emphasized the development of free-flow calibrations or ratings for various flume geometries. Notable free flow investigations have been made by V. M. Cone,[3] Parshall,[4] Engel,[5] Khafagi,[6] Robinson and Chamberlain,[7] and Ackers

Note.—Discussion open until December 1, 1967. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 93, No. HY4, July, 1967. Manuscript was submitted for review for possible publication on June 24, 1966.

[1] Asst. Research Engr., Utah Water Research Lab., College of Engrg., Utah State Univ., Logan, Utah.

[2] Asst. Research Engr., Utah Water Research Lab., College of Engrg., Utah State Univ., Logan, Utah.

[3] Cone, V. M., "The Venturi Flume," Journal of Agricultural Research, Vol. 9, No. 4, April 23, 1917, pp. 115-129.

[4] Parshall, R. L., "The Improved Venturi Flume," Transactions, ASCE, Vol. 89, 1926, pp. 841-851.

[5] Engel, F. V. A. E., "The Venturi Flume," The Engineer, Vol. 158, Aug. 3, 1934, 104-107, Aug. 10, 1934, p. 131-133.

[6] Khafagi, A., "Der Venturikanal. Theorie und Anwendung," No. 1, Mitteilungen aus der Versuchsanstalt fur Wasserbau und Erdbau an der Eidgenossischen Technischen Hochschule in Zurich, Zurich, Gebr. Leeman & Co., 1942.

[7] Robinson, A. R., and Chamberlain, A. R., "Trapezoidal Flumes for Open Channel Flow Measurement," Transactions, American Society of Agricultural Engineers, Vol. 3, No. 2, 1960, pp. 120-124, 128.

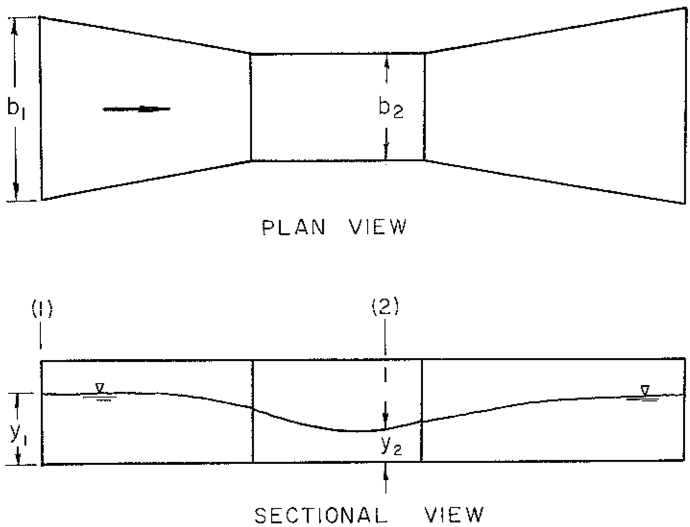

FIG. 1.—DEFINITION SKETCH FOR FLAT-BOTTOMED RECTANGULAR FLUMES

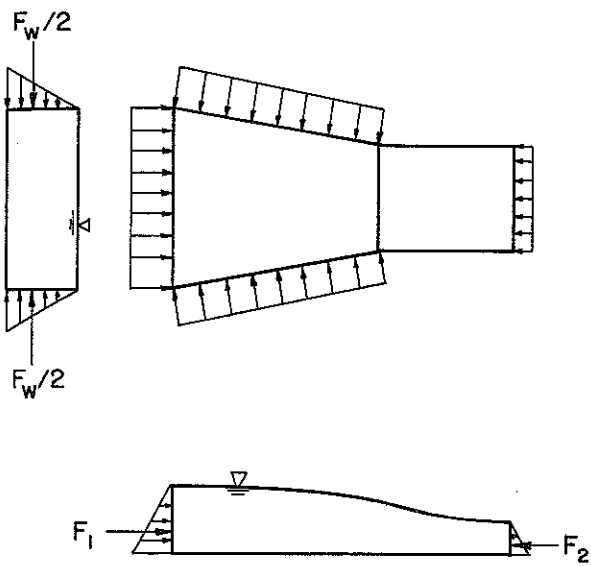

FIG. 2.—CONTROL VOLUME FOR FLAT-BOTTOMED RECTANGULAR FLUMES

and Harrison,[8] to mention a few. Various methods of analyzing submerged flow have been presented by Parshall,[9] Khafagi,[6] Villemonte and Gunaji,[10] Robinson and Chamberlain,[7] and Robinson.[11]

Parameters describing submergence in flow-measuring flumes will be developed from dimensional analysis. A combination of empiricism and dimensional analysis will be used to develop a submerged flow discharge equation. The resulting discharge equation will be compared with the theoretical submerged-flow equation developed from momentum relationships. A rectangular flat-bottomed flow measuring flume was used to generate data necessary for establishing the parameters describing submerged flow. The form of the discharge equation describing submerged flow in a rectangular flume has been verified for a trapezoidal flat-bottomed flume,[12] a rectangular flat-bottomed flume,[13] and a Parshall flume.[14]

MOMENTUM THEORY

A theoretical submerged flow discharge equation will be developed for the flat-bottomed rectangular flume shown in Fig. 1. The momentum equation can be written between sections 1 and 2 for the control volume in Fig. 2 to arrive at a general submerged-flow equation for rectangular flumes. The momentum equation can be written in the direction of flow as

$$F_1 - F_2 - (F_w)_x - F_f = Q_t \rho (\beta_2 V_2 - \beta_1 V_1) \quad \ldots \ldots \quad (1)$$

in which F_1 and F_2 = the resultant forces of the pressure distributions at the two flow cross sections, $(F_w)_x$ = the component of force in the direction of flow acting on the control volume of fluid due to the flume walls, F_f = the friction or drag force acting on the surface of the control volume, Q_t = the theoretical discharge, ρ = the density of the fluid, β_1 and β_2 = momentum coefficients for the two flow sections, and V_1 and V_2 = the average velocity at sections 1 and 2. When uniform velocity distribution is assumed and the friction force is neglected

$$F_1 - F_2 - (F_w)_x = Q_t \rho (V_2 - V_1) \quad \ldots \ldots \quad (2)$$

[8] Ackers, P., and Harrison, A. J. M., "Critical-Depth Flumes for Flow Measurement in Open Channels," Hydraulics Research Paper No. 5, Hydraulics Research Station, Department of Scientific and Industrial Research, Wallingford, Berks., England, 1963.

[9] Parshall, R. L., "Measuring Water in Irrigation Channels with Parshall Flumes and Small Weirs," Circular No. 843, Soil Conservation Service, U. S. Dept. of Agriculture, Washington, D. C., May, 1950.

[10] Villemonte, J. R., and Gunaji, V. N., "Equation for Submerged Sharp-Crested Weirs Found Applicable to 6-inch Parshall Flume," Civil Engineering, Vol. 23, No. 6, June, 1953, pp. 406-407.

[11] Robinson, A. R., "Simplified Flow Corrections for Parshall Flumes Under Submerged Conditions," Civil Engineering, Vol. 35, No. 9, Sept., 1965, p. 75.

[12] Hyatt, M. L., "Design, Calibration, and Evaluation of a Trapezoidal Measuring Flume by Model Study," thesis presented to Utah State University, at Logan, Utah, in 1965, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[13] Skogerboe, G. V., Walker, W. R., and Robinson, L. R., "Design, Operation, and Calibration of the Canal A Submerged Rectangular Measuring Flume," Report PR-WG24-3, Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah, Mar., 1965.

[14] Skogerboe, G. V., Hyatt, M. L., England, J. D., and Johnson, J. R., "Submergence in a Two-Foot Parshall Flume," Report PR-WR6-2, Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah, Aug., 1965.

By assuming hydrostatic pressure distribution

$$F_1 = \frac{\gamma\ b_1\ y_1^2}{2} \qquad (3)$$

and $$F_2 = \frac{\gamma\ b_2\ y_2^2}{2} \qquad (4)$$

in which γ = the specific weight of the fluid, b_1 and b_2 = the width of the flume at sections 1 and 2, and y_1 and y_2 = the depths of flow at the two sections. The force of the flume walls on the control volume acting in the direction of flow occurs in the converging inlet section. The flow depth throughout the converging inlet section is decreasing. A fair approximation, however, of the average flow depth in the section under submerged flow conditions can be assumed as y_1, hence

$$(F_w)_x = \gamma\ (b_1 - b_2)\ \frac{y_1^2}{2} \qquad (5)$$

The momentum equation in the direction of flow can now be written as

$$\frac{\gamma\ b_1\ y_1^2}{2} - \frac{\gamma\ b_2\ y_2^2}{2} - \frac{\gamma\ y_1^2\ (b_1 - b_2)}{2} = \frac{Q_t\ \gamma\ (V_2 - V_1)}{g} \qquad (6)$$

in which g = the acceleration of gravity. By assuming steady flow, the continuity equation, $Q_t = AV$, can be used.

$$Q_t = b_1\ y_1\ V_1 = b_2\ y_2\ V_2 \qquad (7)$$

Substituting the continuity equation into Eq. 6 and solving for the discharge gives

$$Q_t = \frac{(g/2)^{1/2}\ b_2\ (y_1 - y_2)^{1/2}}{\sqrt{\dfrac{[1 - (b_2\ y_2)/(b_1\ y_1)]\ b_2}{b_2\ y_2\ (y_1 + y_2)}}} \qquad (8)$$

Let the constriction ratio, b_2/b_1, be represented by B and the submergence, y_2/y_1, by S. The denominator of the discharge equation can be made dimensionless by multiplying the numerator and denominator by $y_1 - y_2$, so that

$$Q_t = \frac{(g/2)^{1/2}\ b_2\ (y_1 - y_2)^{3/2}}{\sqrt{\dfrac{(1 - BS)\ (y_1 - y_2)^2\ y_1^2}{y_2\ (y_1 + y_2)\ y_1^2}}} \qquad (9)$$

or $$Q_t = \frac{(g/2)^{1/2}\ b_2\ (y_1 - y_2)^{3/2}}{\sqrt{\dfrac{(1 - BS)(1 - S)^2}{S\ (1 + S)}}} \qquad (10)$$

For any particular flume geometry, b_2 and B become constants and the discharge (Eq. 10) is a function of $(y_1 - y_2)^{3/2}$ and S. If the submergence is held constant, the discharge becomes a function of $(y_1 - y_2)^{3/2}$ alone. This suggests that a logarithmic plot of Q against $y_1 - y_2$ would yield a family of

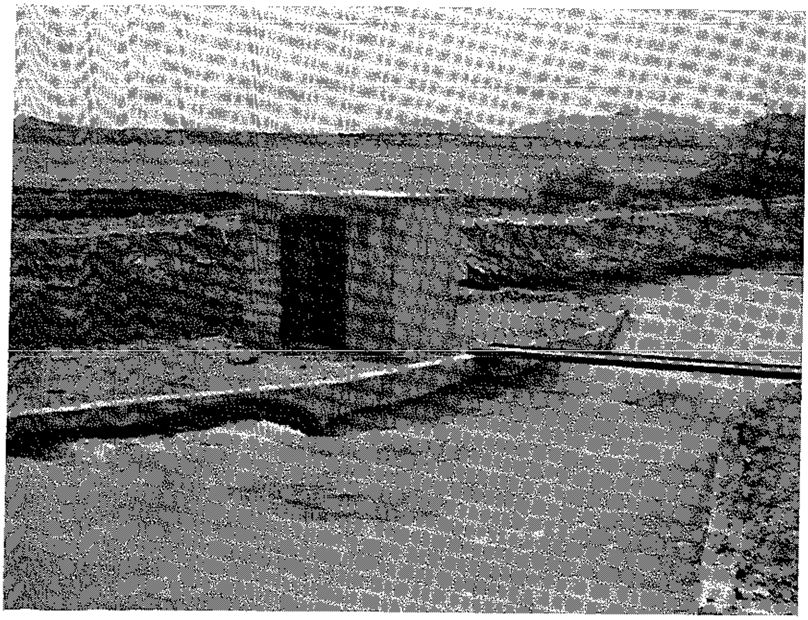

FIG. 3.—RECTANGULAR FLAT-BOTTOMED FLUME WITH DESIGN DISCHARGE OF 500 CFS

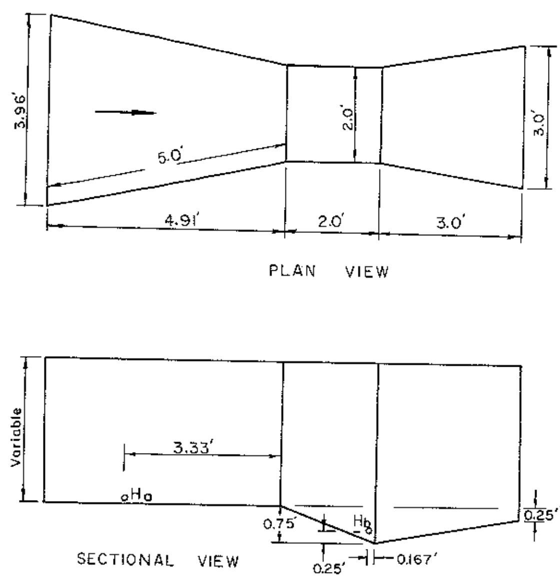

FIG. 4.—DIMENSION DRAWING OF 2-FT. PARSHALL FLUME

Stilling wells were used to measure y_1 and y_3 (Fig. 6). A point gage was used to measure y_m since section m is not stationary. The measurement of y_m is quite difficult because of the fluctuation of the water surface. The average between the fluctuations of the water surface at section m was used in arriving at the value of y_m. The fluctuation was small at low Froude numbers (high submergence), but was pronounced as the Froude number at section m, which will be designated by F_m, approached one.

The free-flow calibration for the rectangular flume is shown in Fig. 8. The free-flow discharge equation as obtained from Fig. 8 is

$$Q = 2.87\, y_1^{1.525} \qquad (13)$$

Dimensional analysis can be used to develop dimensionless parameters describing submerged-flow discharge. Applying such an analysis to the par-

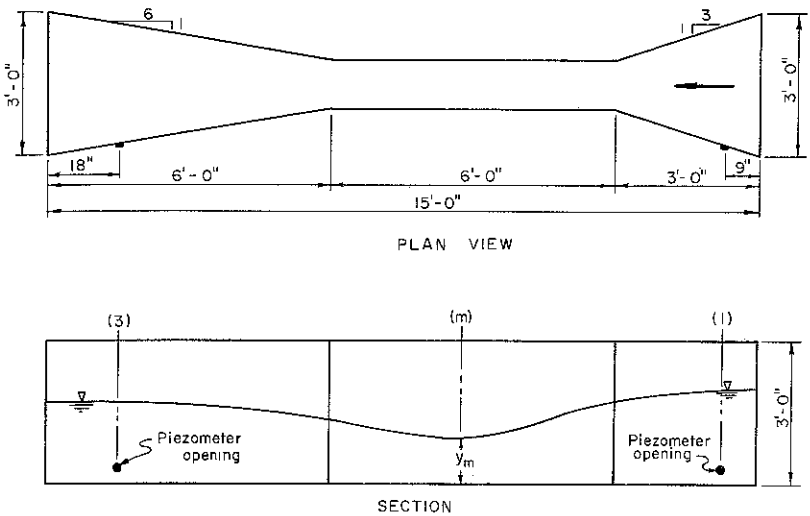

FIG. 7.—DETAILS OF EXPERIMENTAL FLAT-BOTTOMED FLUME

ticular flume geometry under study allows the omission of any geometric terms since they are constant. The variables involved can be written as

$$V = f(g, y_1, y_m, y_3) \qquad (14)$$

With five independent quantities and two dimensions, three pi-terms are necessary.

One of the desired pi-terms is the Froude number, F_m. For a rectangular section, F_m can be expressed by

$$\pi_1 = \mathsf{F}_m = \frac{V}{(g\, y_m)^{1/2}} = \frac{Q}{b_2\, g^{1/2}\, y_m^{3/2}} \qquad (15)$$

in which b_2 = the throat width of the flume.

The second desired pi-term is the submergence, y_3/y_1, which will be designated by S.

$$\pi_2 = S = y_3/y_1 \quad \ldots\ldots (16)$$

The submergence is defined as the ratio of a downstream flow depth to an upstream flow depth, where the downstream measurement is the depth of flow above the flume floor at the point of the upstream measurement. In actuality, the ratio of any flow depth measured downstream from y_m to any flow depth measured upstream from y_m can be used as the submergence, S.

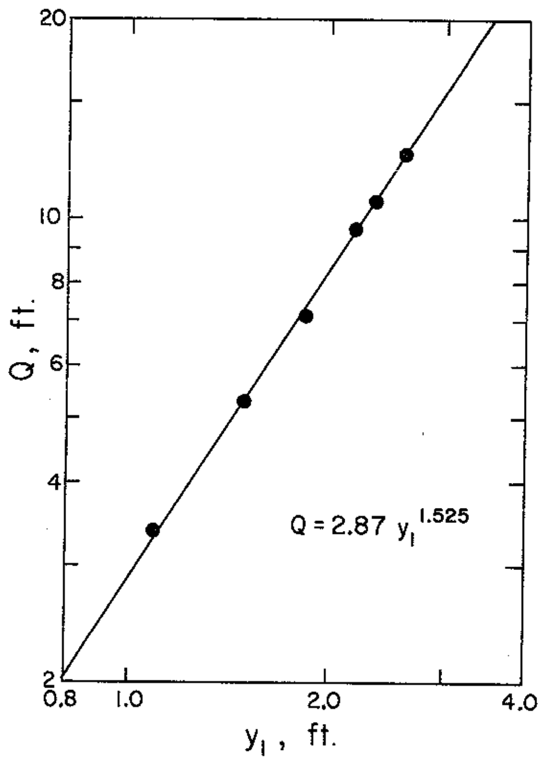

FIG. 8.—FREE FLOW CALIBRATION FOR EXPERIMENTAL FLUME

The third pi-term was developed by trial and error from the trapezoidal flume study.[12] The dimensionless parameter finally developed was

$$\pi_3 = \frac{y_1 - y_3}{y_m} \quad \ldots\ldots (17)$$

From the submerged-flow data, the logarithm of π_2 has been plotted against π_3 in Fig. 9. The curve must pass through the point 0, 0 because as the submergence approaches 100% (log S = 0), the difference in water surface elevation, y_1 - y_3, will approach zero. The relationship between π_2 and π_3 dictates the distribution of the lines of constant submergence in the submerged-flow calibration plot. The relationship in Fig. 9 can be approximated by a straight line over a large range of submergence with some sacrifice in accuracy of the submerged-flow calibration plot. Such an approximation is of value in gaining further insight into the characteristics of submerged flow. The dashed line in Fig. 9 is described by the equation

$$\log S = -0.274 \frac{(y_1 - y_3)}{y_m} - 0.0045 \quad \text{(18)}$$

A logarithmic plot of π_1 against π_3 will yield a straight line relationship which can be combined with Eq. 18 to yield an approximate submerged-flow discharge equation. The resulting equation contains the term $(y_1 - y_3)^{3/2}$

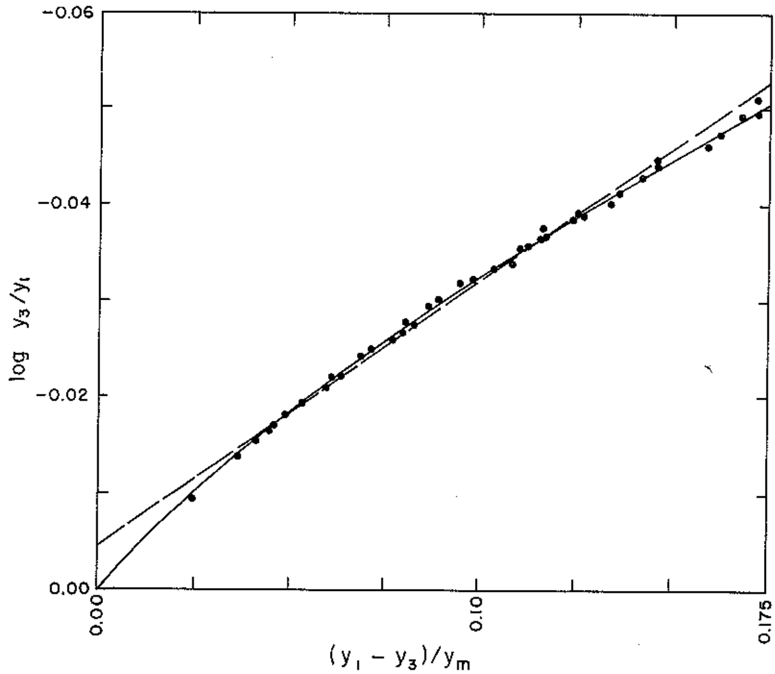

FIG. 9.—RELATIONSHIP BETWEEN π_2 AND π_3

rather than $(y_1 - y_3)^{1.525}$ because the definition of F_m contains $y_m^{3/2}$. To circumvent this situation, Fig. 10 has been prepared with y_m plotted against $y_1 - y_3$ on logarithmic paper with Q as the third variable. An equation relating the three variables has been developed by plotting the value of y_m when $y_1 - y_3$ is equal to one for each line of constant discharge. These intercepts, designated C_m, have been plotted against Q in Fig. 11. The empirical equation resulting from Fig. 10 is

$$y_m = \frac{C_m}{(y_1 - y_3)^{0.425}} \quad \text{(19)}$$

The empirical equation resulting from Fig. 11 is

$$Q = 12.6\ C_m^{1.07} \quad (20)$$

Eqs. 19 and 20 are combined to yield

$$Q = 12.6\ y_m^{1.07}\ (y_1 - y_3)^{0.455} \quad (21)$$

An approximate submerged-flow discharge equation can be obtained by combining Eqs. 18 and 21.

$$Q = \frac{3.15\ (y_1 - y_3)^{1.525}}{[-\ (\log S + 0.0045)]^{1.07}} \quad (22)$$

The dashed lines in Fig. 12 have been obtained from Eq. 22.

The actual submerged-flow calibration plot (solid lines in Fig. 12) can be obtained by combining Eq. 21 with the curve (solid line) in Fig. 9. Comparing

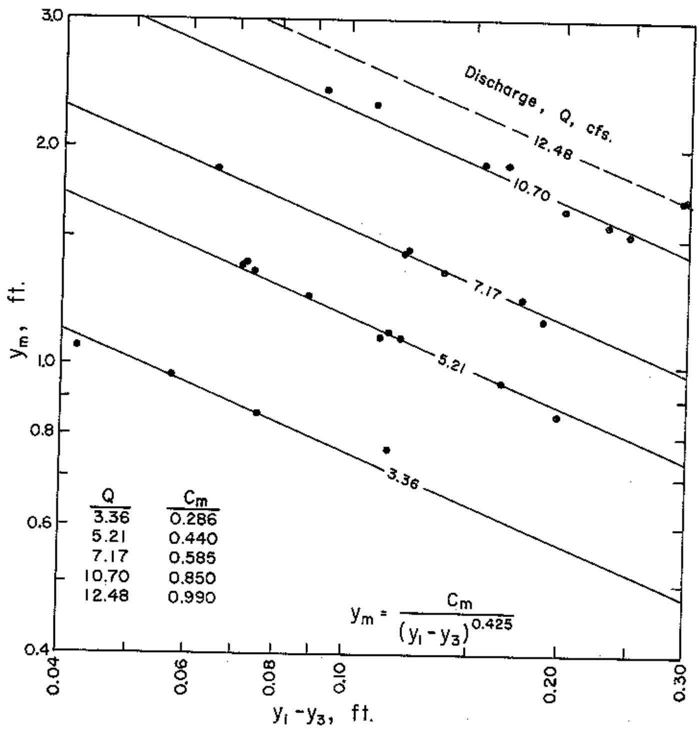

FIG. 10.—PLOT OF MINIMUM FLOW DEPTH, CHANGE IN WATER SURFACE ELEVATION, AND DISCHARGE

the actual calibration plot (solid lines) with the approximate calibration plot (dashed lines) in Fig. 12 indicates the approximate solution (dashed lines) will produce satisfactory results for practical use when the submergence ratio is less than 96%.

The value of submergence at which the transition from free flow to submerged flow occurs can be estimated by equating the free-flow equation and

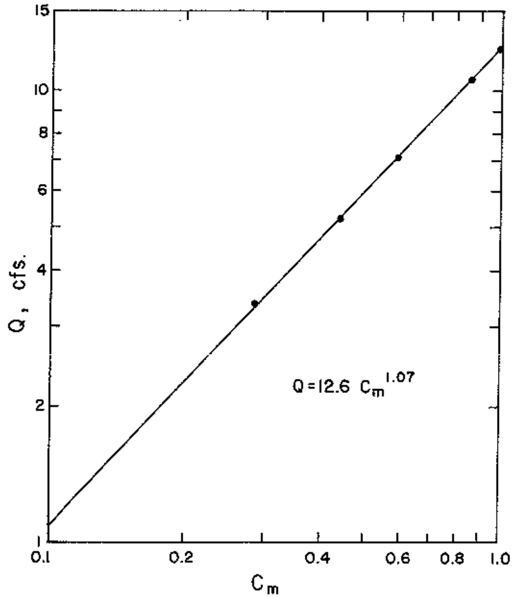

FIG. 11.—DEVELOPMENT OF RELATIONSHIP BETWEEN MINIMUM FLOW DEPTH, CHANGE IN WATER SURFACE ELEVATION, AND DISCHARGE

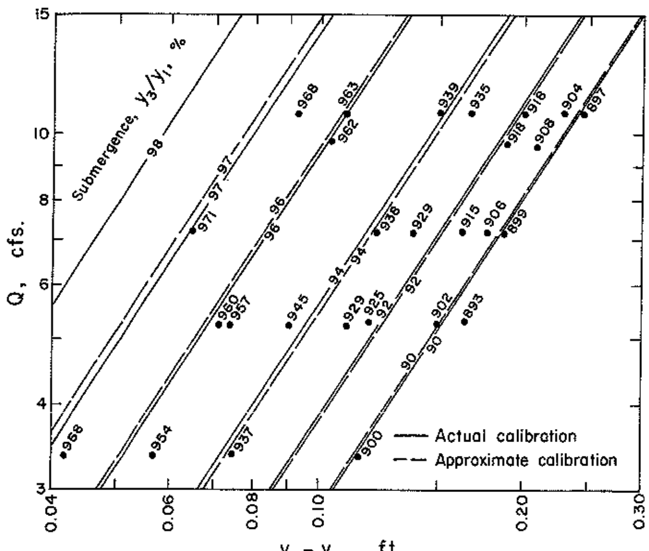

FIG. 12.—SUBMERGED FLOW CALIBRATION CURVES FOR EXPERIMENTAL FLUME

the approximate submerged-flow equation. For the rectangular flume studied, Eqs. 13 and 22 are set equal to one another.

$$\frac{3.15\,(y_1 - y_3)^{1.525}}{[-(\log S + 0.0045)]^{1.07}} = 2.87\,y_1^{1.525} \quad \text{.....................(23)}$$

By trial and error

$$S = 0.893 \quad \text{.....................................(24)}$$

If the free-flow equation (Eq. 13) is equated against the actual submerged-flow calibration (Fig. 9 and Eq. 21), the transition submergence obtained by trial

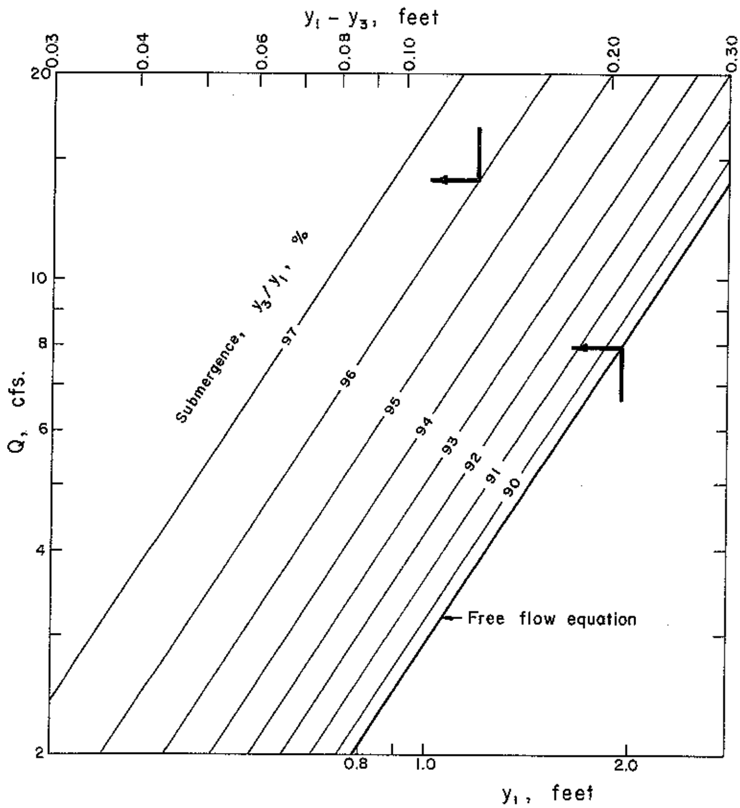

FIG. 13.—FREE FLOW AND SUBMERGED FLOW CALIBRATION FOR EXPERIMENTAL FLUME

and error is 0.896. Hence, free flow exists when the submergence is less than 89.6%, and submerged flow exists for submergences greater than 89.6%. A slight change in the coefficients or powers of the free-flow or submerged-flow equation will have a marked effect on the computation of the transition submergence.

The form of the free-flow and submerged-flow equations allows the calibration curves for both types of flow to be placed on a single chart such as Fig. 13, which constitutes the calibration curves for the rectangular flume studied.

COMPARISONS

The general empirical form of the approximate submerged-flow discharge equation can be obtained from Eq. 22, as

$$Q = \frac{C_1\ (\Delta y)^{n_1}}{[-\ (\log S + C_2)]^{n_2}} \quad \ldots\ldots (25)$$

in which Δy is the change in water surface elevation between a section upstream and one downstream from section m. An appraisal of Eqs. 10 and 25 discloses some similarities. For a particular rectangular flume with specified dimensions, the contraction ratio, B, and throat width, b_2, are constants.

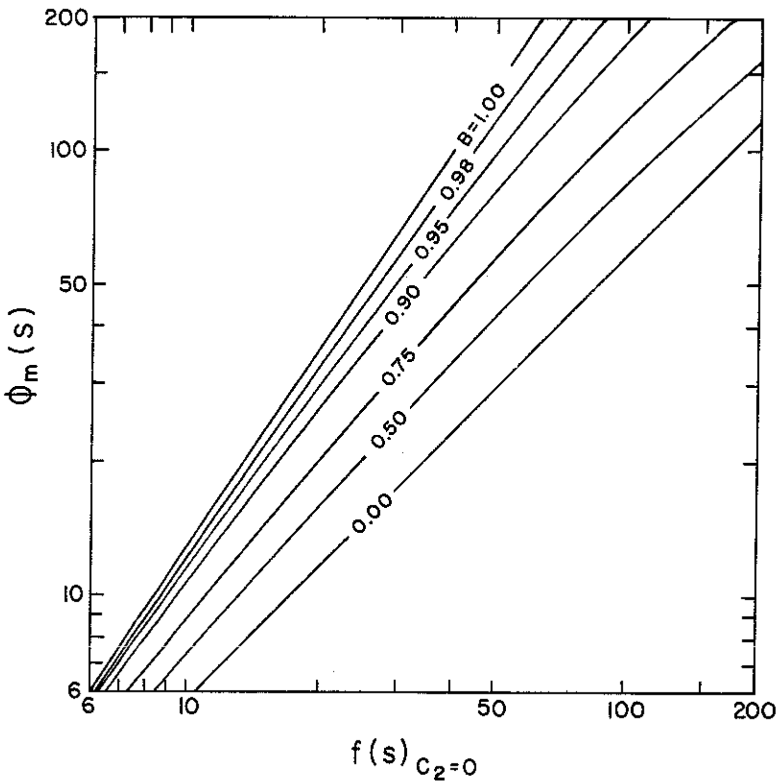

FIG. 14.—RELATIONSHIP BETWEEN $\phi_m(S)$ AND $f(S)_{C_2=0}$

Consequently, the theoretical discharge, Q_t, becomes a function of $(y_1 - y_2)^{3/2}$ and S, which is quite similar to the results obtained empirically where the discharge is a function of $(y_1 - y_2)^{n_1}$ and S. The value of n_1 for rectangular flumes is greater than 3/2 as indicated by Eq. 13, and the n_1 values for all Parshall flumes[4] are slightly in excess of 3/2, ranging in value from 1.52 to 1.60.

The similarity between the denominators of Eqs. 10 and 25 can be shown by letting

$$\phi_m (S) = \frac{1}{\sqrt{\dfrac{(1 - BS)(1 - S)^2}{S(1 + S)}}} \qquad (26)$$

and $$f(S) = \frac{1}{-(\log S + C_2)} \qquad (27)$$

The value of C_2 in Eqs. 25 and 27 can be obtained from Fig. 9 by drawing a line tangent to the distribution curve for a given submergence value, and

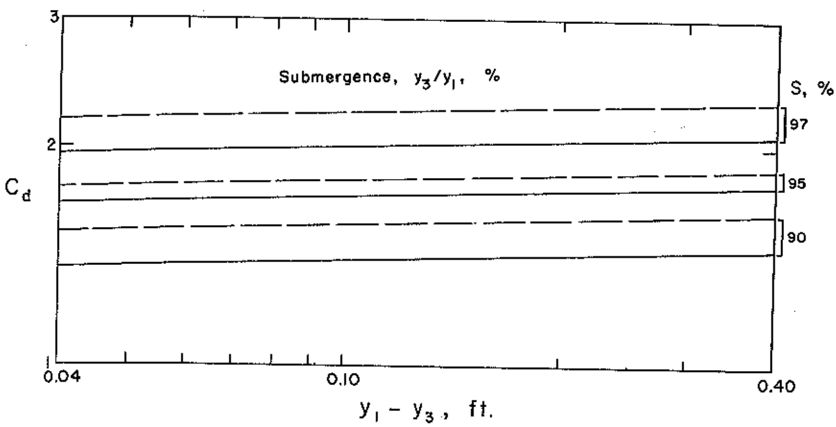

FIG. 15.—COEFFICIENT OF DISCHARGE FOR EXPERIMENTAL FLUME

reading the intercept from the ordinate. The value of C_2 decreases to zero as the submergence ratio increases to one. When C_2 does equal zero, Eq. 27 can be written as

$$f(S)_{C_2=0} = \frac{1}{-\log S} \qquad (28)$$

The relation between $\phi_m(S)$ and $f(S)_{C_2=0}$ is bounded by the constriction ratios 0.0 and 1.0 as shown in Fig. 14. The slope of each line of constant constriction ratio, which is the exponent n_{2t}, ranges in value from 1.0 to 1.5 for corresponding B values of 0.0 and 1.0, respectively. The nomenclature n_{2t} is used to signify the value of n_2 derived by comparison with the pressure-momentum equation (Eq. 10) where $f(S)$ is evaluated for $C_2 = 0$. The curves depicted in Fig. 14 can be written in equation form as

$$\phi_m(S) = C_3\, f(S)_{C_2=0}^{n_{2t}} = \frac{C_3}{[-(\log S)]^{n_{2t}}} \qquad (29)$$

The relation between $\phi_m(S)$ and $f(S)_{C_2=0}$ is not quite so simple for constriction ratios other than 0.0 and 1.0 since the lines of constant constriction ratio show a slight curvature in Fig. 14.

A common method for utilizing a theoretical discharge equation to fit actual data is to use a coefficient of discharge, C_d, which is defined as the ratio of the actual discharge to the theoretical discharge,

$$C_d = \frac{Q}{Q_t} \qquad (30)$$

The submerged flow calibration plot for the experimental rectangular flume (Fig. 12) has been used in conjunction with the theoretical submerged flow discharge equation (Eq. 10) to develop the coefficient of discharge relation for the experimental flume (Fig. 15). For any flume, C_d is a function of both Δy and S. Because finite values of both Δy and S result in a unique solution of both flow depths, C_d varies as the flow depths vary. The coefficient of discharge for any flume becomes a three-dimensional plot similar in form to the submerged-flow calibration curves. The slope of the lines of constant submergence in the plot of C_d is $n_1 - 3/2$. For example, the slope of the lines in Fig. 15 is 0.025 (1.525 - 3/2).

CONCLUSIONS

The general form of the free-flow calibration equation for a flume can be written as

$$Q = C\, y_1^{n_1} \qquad (11)$$

The analysis of data for a rectangular flume has led to the development of an approximate submerged-flow equation which can be written in general form as

$$Q = \frac{C_1 (\Delta y)^{n_1}}{[-(\log S + C_2)]^{n_2}} \qquad (25)$$

The use of Eq. 25 has been found to be valid for a trapezoidal flat-bottomed flume,[12] a rectangular flat-bottomed flume,[13] and a Parshall flume.[14] Of particular significance is that the exponent, n_1, appears both in the free-flow equation and the submerged-flow equation. Because of this, the free-flow and submerged-flow equations developed for any particular flume geometry can be set equal to one another, and the solution will yield the value of submergence at which the transition from free flow to submerged flow occurs. Also, since n_1 appears in both equations, calibration curves for both types of flow can be placed on a single graph.

The submerged-flow equation developed from momentum relationships for a rectangular flume is

$$Q_t = \frac{(g/2)^{1/2}\ b_2\ (y_1 - y_2)^{3/2}}{\sqrt{\frac{(1 - BS)(1 - S)^2}{S\ (1 + S)}}} \qquad (10)$$

When Eqs. 25 and 10 are compared for a rectangular flume, the exponent, n_1, in Eq. 25 is found to exceed the theoretical value of 3/2. The denominators of the two equations are compatible if C_2 equals zero. The empirical analysis does show that C_2 approaches zero as the submergence, S, approaches one. The comparison of the denominators of the two equations showed that n_2 varies between 1.0 and 1.5.

ACKNOWLEDGMENTS

The work upon which this paper is based was supported in part from funds provided by the United States Department of the Interior as authorized under the Water Resources Research Act of 1964, Public Law 88-379. The project was conducted as a part of the program of the Utah Center for Water Resources Research, Utah State University, Logan, Utah.

APPENDIX.—NOTATION

The following symbols are used in this paper:

A = cross-sectional area of flow;
A_1 = cross-sectional area at entrance of flume;
A_2 = cross-sectional area at throat of flume;
A_m = minimum cross-sectional flow area in the throat;
B = constriction ratio, b_2/b_1;
b = bottom width of trapezoidal flume throat;
b_1 = bottom width at entrance of rectangular flume;
b_2 = bottom width at throat of rectangular flume;
C = coefficient in the free flow equation;
C_1 = coefficient in the numerator of the submerged flow equation;
C_2 = coefficient in the denominator of the submerged flow equation;
C_3 = coefficient relating $\phi_m(S)$ and $f(S)$;
C_d = coefficient of discharge;
C_m = coefficient relating discharge, minimum flow depth in throat, and change in water surface elevation;
F_1 = hydrostatic force at section 1;
F_2 = hydrostatic force at section 2;
F_f = frictional or drag force;
F_m = maximum Froude number in the throat;
F_w = hydrostatic force on flume walls in entrance section;
$(F_w)_x$ = hydrostatic force on flume walls in entrance section acting in the direction of flow;
f = function;

g = acceleration of gravity;
H_a = flow depth in entrance section of Parshall flume;
H_b = flow depth in throat of Parshall flume;
n_1 = power of y_1 in the free flow equation;
n_2 = power of the submergence term in the denominator of the submerged flow equation;
n_{2t} = value of n_2 derived from relationship between $\phi_m(S)$ and $f(S)$;
Q = flow rate or discharge;
Q_t = theoretical discharge;
S = submergence; ratio of a downstream flow depth to an upstream flow depth, where the downstream measurement is the depth of flow above the flume floor at the point of upstream measurement;
V = average velocity;
V_1 = average velocity at section 1;
V_2 = average velocity at section 2;
y_1 = flow depth at section 1;
y_2 = flow depth at section 2;
y_3 = flow depth in exit section of experimental rectangular flat-bottomed flume;
y_m = minimum depth of flow in flume throat, which varies in location longitudinally;
Δy = change in water surface elevation between a section upstream and one downstream from section m;
π_1 = F_m;
π_2 = S;
π_3 = $\Delta y/y_m$;
β = momentum correction coefficient;
γ = specific weight of fluid;
ρ = density of fluid;
$f(S)$ = defined by $1/(-\log S)$; and
$\phi_m(S)$ = defined by $1/[(1 - BS)(1 - S)^2/S(1 + S)]^{1/2}$.

5348 SUBMERGENCE IN FLOW MEASURING FLUMES

KEY WORDS: channels (waterways); flow measurement; fluid flow; hydraulics; subcritical flow; submerged flow

ABSTRACT: The calibration curves which describe submergence in flow-measuring flumes are developed by a combination of dimensional analysis and empiricism. The parameters developed in this manner are further verified by the theoretical submerged flow equation developed from momentum relationships. A flat-bottomed rectangular measuring flume was used to generate data necessary for establishing the parameters describing submerged flow. The resulting form of the discharge equation has been verified for a trapezoidal flat-bottomed flume and a Parshall flume. For any particular flume geometry, both the free flow and submerged flow equations can be placed on a single graph.

REFERENCE: Skogerboe, Gaylord V., and Hyatt, M. Lyon, "Analysis of Submergence in Flow Measuring Flumes," Journal of the Hydraulics Division, ASCE, Vol. 93, No. HY4, Proc. Paper 5348, July, 1967, pp. 183-200.